A question that combines many aspects of planetary physics.

Questions:

a) What is the definition of a Dwarf Planet?

b) Draw a labelled diagram of the orbit of a planet having a high eccentricity and give the names of the points on the orbit where the planet is furthest from and closest to the Sun.

c) The newly discovered Dwarf Planet, Eris, has an orbit whose period is 560 years. The eccentricity of its orbit is 0.3. Calculate how close, in AU, it will approach the Sun. (The eccentricity is given by the distance between the foci of the elliptical orbit divided by its major axis.)

d) Eris has a satellite, Dysnomia, which orbits Eris in a circular orbit once every 15.77 days at a distance of 37,350 km. This allows the mass of Eris to be calculated:

i) Show, by equating the gravitational force between them and the apparent force

due to centripetal acceleration that:

$$M_{Eris} = v^2_{dysnomia} \ a / G$$

(Where a is the radius of Dysnomia's orbit and G the Universal Constant of Gravitation whose value is 6.67 x 10^{-11} N m^2/kg^2)

ii) Calculate Dysnomia's orbital velocity in m s^{-1}.

iii) Hence calculate the mass of Eris in kg.

Answers:

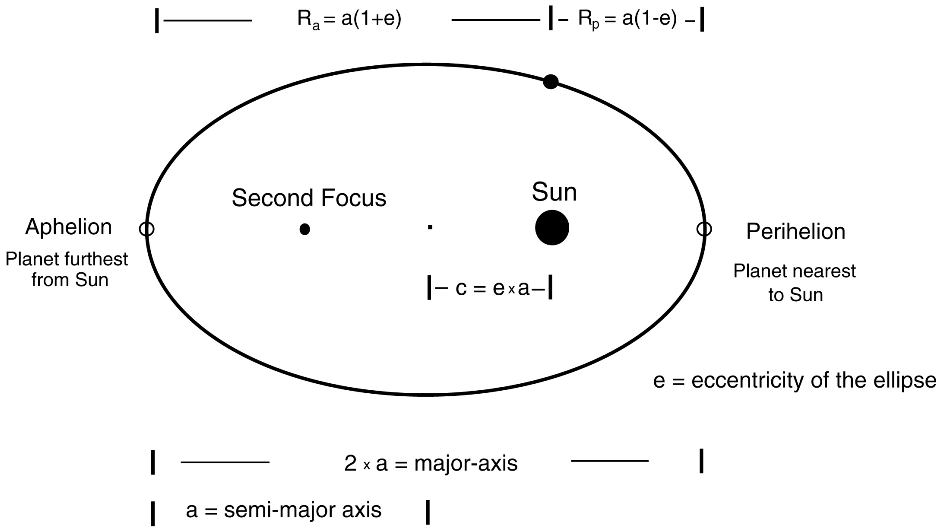
a)

(i) Has enough mass so that gravity can overcome the strength of the body and so becomes approximately round. It is said to be in hydrostatic equilibrium.

(ii) It has NOT, however, "cleared" its orbit. That is, it is NOT the only body of its size in the region of the Solar System at that distance from the Sun.

(ii) It is NOT a satellite of a larger body.

b)



c) Use Kepler's Third Law to find the semi-major axis:

$$a = (560)^{2/3} \quad = \quad 67.9 \text{ AU}$$

The major axis is thus 135.9 AU.
The distance between the foci is the major axis multiplied by the eccentricity = 40.8 AU.

The nearest that Eris will approach the Sun is the (semi major axis – half the distance between the foci), that is: 67.9 – 20.4 = 47.5 AU.

d)

The gravitational force between Mars and Eris is given by $M_{Eris}\, m_{Dysnomia}\, G / a^2$ where, a is the semi-major axis of Eris's orbit and G the Universal constant of gravitation.

This force must equal that resulting from the centripetal acceleration¸ ω^2, or v^2/a on Eris so :

$$M_{Eris}\, m_{Dysnomia}\, G / a^2 \qquad = m_{Dysnomia}\, v^2_{\ Dysnomia} / a$$

Giving:

$$M_{Eris} = v^2_{\ Dysnomia}\, a / G$$

But v = 2 π a/P where P is the period of Dysnomia (in seconds) giving:

$$v_{Dysnomia} = 2 \times 3.14159 \times 37{,}350{,}000 / 15.77 \times 86{,}400$$

$$= 172.2 \text{ m s}^{-1}$$

$$M_{Eris} = (172.2)^2 \times 37{,}350{,}000 / 6.67 \times 10^{-11}$$

$$= 1.66 \times 10^{22} \text{ kg}$$

(This IS precisely the currently accepted mass of Eris)